Power scaling rules for charmonia production and HQEFT [*]

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Abstract

We discuss the power scaling rules along the lines of a complete Heavy Quark Effective Field Theory (HQEFT) for the description of heavy quarkonium production through a color-octet mechanism. To this end, we firstly derive a tree-level heavy quark effective Lagrangian keeping both particle-antiparticle mixed sectors allowing for heavy quark-antiquark pair annihilation and creation, but describing only low-energy modes around the heavy quark mass. Then we show the consistency of using HQEFT fields in constructing four-fermion local operators *à la NRQCD*, to be identified with standard color-octet matrix elements. We analyze some numerical values extracted from charmonia production by different authors and their hierarchy in the light of HQEFT.

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1   Introduction

The application of effective theories directly derived from first principles - Quantum Chromodynamics (QCD) in particular - to heavy flavor physics has currently become a very important tool to cope with the complexity of the strong interaction dynamics. One of its advantages is the appearance of symmetries in the very large quark mass limit, not manifest in the full theory. Moreover, symmetry breaking effects due to the finiteness of quark masses can be systematically incorporated via power expansions in the reciprocal of the heavy quark mass. One of the foremost applications of heavy-quark spin-flavor symmetry has led to accurate and almost model-independent extractions of some Kobayashi-Maskawa matrix elements in inclusive and exclusive B meson decays (see [1] for a review).

Over the nineties, a heavy quark effective theory (HQET) was developed [2, 3] to deal with the phenomenology of heavy-light mesons and baryons. Moreover, there were early attempts to extend its initial scope to heavy-heavy bound states like the J/ψ resonance [4] or the B_c meson and doubly heavy baryons [5]. On the other hand, a close approach also emerged in parallel, especially designed to describe heavy quarkonia inclusive decay and production, the Non-Relativistic QCD (NRQCD)[6, 7]. In this formalism, the decay widths or cross sections are factorized as a sum of products of short-distance coefficients and long-distance NRQCD matrix elements.

Although both HQET and NRQCD Lagrangians present a formal analogy, there is a basic difference between both frameworks. In the former one, there are basically two relevant scales: the heavy quark mass m_Q and Λ_{QCD}, the characteristic parameter of the strong interaction. In NRQCD, besides Λ_{QCD}, there are (at least) another two low-energy scales: $m_Q v'$ (soft) and $m_Q v'^2$ (ultrasoft) [1].

Recently, several applications of HQET involving both heavy quark and antiquark fields altogether to the study of the phenomenology of the weak and strong interactions have appeared in the literature ([8, 9] and [10], respectively). At the same time it becomes suitable a careful survey of the foundations on which such a theoretical framework is based. Therefore, in this paper firstly we shall focus on the quantum aspects of the heavy quark effective fields, summarizing some work presented in previous references [11, 12]. In particular, we shall keep both particle-antiparticle mixed sectors of the effective Lagrangian, in accordance with the pioneering work by Wu in Ref. [13], further developed and discussed in [14]. Hereafter we shall refer to this extended framework as HQEFT as in [8], to distinguish it from "usual" HQET.

Furthermore, one of the goals of this paper is to analyze whether HQEFT could provide an appropriate framework to describe meaningfully the hadroproduction of charmonium. In particular, we shall analyze the power counting of four-fermion local operators, whose scaling rules for charmonia have been recently hypothesized in Ref. [15] along the lines of HQET. We shall examine this issue with the help of diverse extractions of some matrix elements performed by several authors.

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[1]In this paper we shall employ v' to denote the three-velocity of heavy quarks inside quarkonium while v will stand for the hadron four-velocity, as usual in HQET; since residual momenta of heavy quarks will not be neglected in the hadron rest frame, v' obviously does not exactly coincide with the spatial component of v, i.e. $v' \neq |\mathbf{v}|$.

2 Notation and definitions

Let us start following Ref. [11] by writing the plane wave Fourier expansion for a fermionic field $Q_v(x)$ corresponding to an almost on-shell heavy quark or antiquark (we omit indices relative to flavor and color) moving inside a hadron with four-velocity v,

$$Q_v(x) = Q_v^{(+)}(x) + Q_v^{(-)}(x) = \int \frac{d^3\mathbf{p}}{J} \sum_r [\, b_r(\mathbf{p})\, u_r(\mathbf{p})\, e^{-ip\cdot x} + \tilde{b}_r^\dagger(\mathbf{p})\, v_r(\mathbf{p})\, e^{ip\cdot x}\,] \tag{1}$$

where r refers to the spin and J stands for the chosen normalization; $b_r(\mathbf{p})/\tilde{b}_r^\dagger(\mathbf{p})$ is the annihilation/creation operator for a heavy quark/antiquark with three-momentum $\mathbf{p}$ ($p^0 \simeq + \sqrt{m^2 + \mathbf{p}^2}$). Let us firstly focus on the particle sector of the theory.

Particle Sector

As the heavy quark is almost on-shell one should require for *each Fourier component* that $p^2 = m_Q^2 + \Delta^2$ where Δ^2 satisfies $\lim_{m_Q \to \infty} \Delta^2/m_Q^2 \to 0$. From now on, we shall set $\Delta^2 = 0$ as a good approximation for heavy quarks bound in hadrons.

On the other hand, spinors are normalized such that $u_r^\dagger(\mathbf{p})u_s(\mathbf{p}) = 2p^0 N\, \delta_{rs}$ and the creation/annihilation operators satisfy:

$$[b_r(\mathbf{p}'), b_s^\dagger(\mathbf{p})]_+ = K\, \delta_{rs}\, \delta^3(\mathbf{p}' - \mathbf{p}) \tag{2}$$

where K, N are the corresponding normalization factors.

Now let us redefine the momentum of each Fourier component in Eq. (1) according to HQET as the sum of a mechanical part and a *Fourier residual four-momentum k*,

$$p = m_Q v + k \tag{3}$$

Hence one can write from (1)

$$Q_v^{(+)}(x) = e^{-im_Q v\cdot x} \int \frac{d^3\mathbf{k}}{J} \sum_r b_r(v, \mathbf{k})\, u_r(\mathbf{k})\, e^{-ik\cdot x} \tag{4}$$

The main point to be stressed again is that we shall require that each Fourier component should satisfy the almost on-shell condition $p_Q^2 \simeq m_Q^2$. Therefore,

$$k^2 + 2m_Q v\cdot k = 0 \tag{5}$$

Thus, in the hadron rest frame, the kinetic energy of the heavy quark $v\cdot k = k^0$ is related to $\mathbf{k}$ through the constraint

$$(k^0)^2 + 2m_Q k^0 - \mathbf{k}^2 = 0 \tag{6}$$

which yields the expected relation for the positive root $k^0 = \sqrt{m_Q^2 + \mathbf{k}^2} - m_Q$. In the non-relativistic limit obviously, $k^0 \simeq \mathbf{k}^2/2m_Q$.

Notice that the annihilation (and creation) operators and spinors have been simply *relabeled* in Eq. (4), satisfying the same normalization as above, though expressed in terms of the Fourier residual momentum $\mathbf{k}$. In particular, $b_r(v, \mathbf{k})/b_r^\dagger(v, \mathbf{k})$ correspond to annihilation/creation operators of a heavy quark with residual momentum $\mathbf{k}$ in a hadron moving with four-velocity v, satisfying accordingly

$$[b_r(v, \mathbf{k}'), b_s^\dagger(v, \mathbf{k})]_+ = K \; \delta_{rs} \; \delta^3(\mathbf{k}' - \mathbf{k}) \tag{7}$$

and

$$\sum_r u_r(\mathbf{k})\bar{u}_r(\mathbf{k}) \;=\; N \, [\, m_Q(1 + \slashed{v}) + \slashed{k} \,] \tag{8}$$

where the normalization factors must obey the combined relation: [16] [2]

$$I \;=\; \frac{K\,N}{J^2} \;=\; \frac{1}{(2\pi)^3 \, 2p^0} \;=\; \frac{1}{(2\pi)^3 \, 2(m_Q v^0 + k^0)} \tag{9}$$

On the other hand, it is quite usual in the literature to identify effective heavy quark fields with those "leading" components of the Fourier expansion corresponding to momenta $p^\mu \simeq m_Q v^\mu$ (or equivalently with k^μ components close to zero), so they rather look like single spinors or anti-spinors at leading order in $1/m_Q$. However, this is a too restrictive view since in constructing HQEFT one should allow the $\mathbf{k}$ components of the effective quantum fields to range over values of the order of Λ_{QCD}.

Next let us introduce the effective fields in the following standard manner [3] [17],

$$h_v^{(+)}(x) \;=\; e^{im_Q v \cdot x} \frac{1 + \slashed{v}}{2} \, Q_v^{(+)}(x) \tag{10}$$

$$H_v^{(+)}(x) \;=\; e^{im_Q v \cdot x} \frac{1 - \slashed{v}}{2} \, Q_v^{(+)}(x) \tag{11}$$

where $H_v^{(+)}(x)$ represents the "small" (lower in the hadron reference frame) component of the corresponding spinor field. Thus one may identify from the expansion (4)

$$h_v^{(+)}(x) \;=\; \frac{1 + \slashed{v}}{2} \int \frac{d^3\mathbf{k}}{J} \sum_r b_r(v, \mathbf{k}) \, u_r(\mathbf{k}) \, e^{-ik \cdot x} \tag{12}$$

$$H_v^{(+)}(x) \;=\; \frac{1 - \slashed{v}}{2} \int \frac{d^3\mathbf{k}}{J} \sum_r b_r(v, \mathbf{k}) \, u_r(\mathbf{k}) \, e^{-ik \cdot x} \tag{13}$$

Sometimes it can be read in the literature that $h_v^{(+)}$ annihilates a heavy quark whereas allegedly $H_v^{(+)}$ creates a heavy antiquark, both with velocity v. Although $H_v^{(+)}$ may take into account the antiparticle features of a relativistic fermion, its Fourier decomposition does not allow for such a process, as can be seen from (13).

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[2] Note that $I \neq 1/(2\pi)^3 2k^0$ since k and p are not related through a Lorentz transformation; indeed, Eq. (3) represents an energy-momentum shift, to get rid of the unwanted frequency modes to be subsequently eliminated in the effective theory.

3 Anti-particle Sector

Proceeding in a parallel way as in the particle sector, let us remark however that now the Fourier expansion of $Q_v^{(-)}(x)$ involves negative frequencies. Therefore, starting from Eq. (1) we introduce the Fourier residual momentum in this case as

$$p = m_Q v - k \tag{14}$$

so k^0 will explicitly exhibit its negative sign. In effect, let us write

$$Q_v^{(-)}(x) = e^{im_Q v \cdot x} \int \frac{d^3\mathbf{k}}{J} \sum_r \tilde{b}_r^\dagger(\mathbf{k}) \, v_r(\mathbf{k}) \, e^{-ik \cdot x} \tag{15}$$

The slightly off-shellness condition $p^2 \simeq m_Q^2$ now implies from Eq. (14):

$$k^2 - 2m_Q v \cdot k = 0 \tag{16}$$

which can be written in the frame moving with velocity v as

$$(k^0)^2 - 2m_Q k^0 - \mathbf{k}^2 = 0 \tag{17}$$

whose negative root reads $k^0 = m_Q - \sqrt{m_Q^2 + \mathbf{k}^2} \simeq -\mathbf{k}^2/2m_Q$.

The effective fields are then defined as

$$h_v^{(-)}(x) = e^{-im_Q v \cdot x} \frac{1 - \slashed{v}}{2} Q_v^{(-)}(x) = \frac{1 - \slashed{v}}{2} \int \frac{d^3\mathbf{k}}{J} \sum_r \tilde{b}_r^\dagger(\mathbf{k}) \, v_r(\mathbf{k}) \, e^{-ik \cdot x} \tag{18}$$

$$H_v^{(-)}(x) = e^{-im_Q v \cdot x} \frac{1 + \slashed{v}}{2} Q_v^{(-)}(x) = \frac{1 + \slashed{v}}{2} \int \frac{d^3\mathbf{k}}{J} \sum_r \tilde{b}_r^\dagger(\mathbf{k}) \, v_r(\mathbf{k}) \, e^{-ik \cdot x} \tag{19}$$

where the anti-spinors satisfy

$$\sum_r v_r(\mathbf{k}) \bar{v}_r(\mathbf{k}) = N \, [\, m_Q(\slashed{v} - 1) - \slashed{k} \,] \tag{20}$$

4 A complete tree-level HQEFT Lagrangian

Once introduced the notation and definitions for the heavy-quark effective fields in the previous Section, we actually get started by expressing the Lagrangian in terms of the effective fields keeping all non-null terms, leading in principle to the possibility of describing annihilation or creation of heavy quark-antiquark pairs at tree level. Indeed after performing an energy-momentum shift by introducing a center-of-mass residual momentum, only low energy modes of the fields (about the heavy quark mass) would be involved making meaningful our approach within the framework of an effective theory.

Thereby a crucial difference of this approach w.r.t. other standard works is that we are concerned with the existence of two-fermion terms in the transformed Lagrangian mixing large components of the heavy quark and heavy antiquark fields, i.e. $h_v^{(\pm)}\Gamma h_v^{(\mp)}$, where Γ stands for a combination of Dirac gamma matrices and covariant derivatives. The implications on the description of heavy quarkonia production will be seen in Section 6.

The tree-level QCD Lagrangian is our point of departure:

$$\mathcal{L} = \overline{Q}\,(i\overrightarrow{\slashed{D}} - m_Q)\,Q \tag{21}$$

where

$$Q = Q^{(+)} + Q^{(-)} = e^{-im_Q v\cdot x}\left[h_v^{(+)} + H_v^{(+)}\right] + e^{im_Q v\cdot x}\left[h_v^{(-)} + H_v^{(-)}\right] \tag{22}$$

and D standing for the covariant derivative

$$\overrightarrow{D}^\mu = \overrightarrow{\partial}^\mu - igT_a A_a^\mu$$

with T_a the generators of $SU(3)_c$. Substituting (22) into (21) one easily arrives at

$$\mathcal{L} = \mathcal{L}^{(++)} + \mathcal{L}^{(--)} + \mathcal{L}^{(-+)} + \mathcal{L}^{(+-)} \tag{23}$$

where we have explicitly splitted the Lagrangian into four different pieces corresponding to the particle-particle, antiparticle-antiparticle and both particle-antiparticle sectors. The former one has the form

$$\mathcal{L}^{(++)} = \bar{h}_v^{(+)}iv\cdot\overrightarrow{D}h_v^{(+)} - \bar{H}_v^{(+)}(iv\cdot\overrightarrow{D} + 2m_Q)H_v^{(+)} + \bar{h}_v^{(+)}i\overrightarrow{\slashed{D}}_\perp H_v^{(+)} + \bar{H}_v^{(+)}i\overrightarrow{\slashed{D}}_\perp h_v^{(+)} \tag{24}$$

corresponding to the usual HQET Lagrangian still containing both $h_v^{(+)}$ and $H_v^{(+)}$ fields. We employ the common notation where perpendicular indices are implied according to

$$D_\perp^\mu = D_\alpha\,(g^{\mu\alpha} - v^\mu v^\alpha)$$

Regarding the antiquark sector of the theory

$$\mathcal{L}^{(--)} = -\bar{h}_v^{(-)}iv\cdot\overrightarrow{D}h_v^{(-)} + \bar{H}_v^{(-)}(iv\cdot\overrightarrow{D} - 2m_Q)H_v^{(-)} + \bar{h}_v^{(-)}i\overrightarrow{\slashed{D}}_\perp H_v^{(-)} + \bar{H}_v^{(-)}i\overrightarrow{\slashed{D}}_\perp h_v^{(-)} \tag{25}$$

The latter expressions, considered as quantum field Lagrangians, certainly do not afford tree-level heavy quark-antiquark pair creation or annihilation processes stemming from the terms mixing $h_v^{(\pm)}$ and $H_v^{(\pm)}$ fields since they contain either annihilation and creation operators of heavy quarks or annihilation and creation operators of heavy antiquarks separately.

Nevertheless there are two extra pieces in the Lagrangian (23):

$$\mathcal{L}^{(-+)} = e^{-i2m_Q v\cdot x} \times \tag{26}$$
$$[\,\bar{H}_v^{(-)}iv\cdot\overrightarrow{D}h_v^{(+)} - \bar{h}_v^{(-)}(iv\cdot\overrightarrow{D} + 2m_Q)H_v^{(+)} + \bar{h}_v^{(-)}i\overrightarrow{\slashed{D}}_\perp h_v^{(+)} + \bar{H}_v^{(-)}i\overrightarrow{\slashed{D}}_\perp H_v^{(+)}\,]$$

and

$$\mathcal{L}^{(+-)} = e^{i2m_Q v\cdot x} \times$$
$$[\, -\bar{H}_v^{(+)} iv\cdot\overrightarrow{D}\, h_v^{(-)} + \bar{h}_v^{(+)}(iv\cdot\overrightarrow{D} - 2m_Q)H_v^{(-)} + \bar{h}_v^{(+)} i \overrightarrow{\slashed{\mathcal{P}}}_\perp h_v^{(-)} + \bar{H}_v^{(+)} i \overrightarrow{\slashed{\mathcal{P}}}_\perp H_v^{(-)} \,] \tag{27}$$

where use was made of the orthogonality of the $h_v^{(\pm)}$ and $H_v^{(\pm)}$ fields [14]. As could be expected, there are indeed pieces mixing both quark and antiquark fields leading to the possibility of annihilation/creation processes. After all, Lagrangian (23) is still equivalent to full (tree-level) QCD [3].

Let us remark that, at first sight, one might think that the rapidly oscillating exponentials in Eqs. (26-27) would make both $\mathcal{L}^{(-+)}$ and $\mathcal{L}^{(+-)}$ pieces to vanish, once integrated over all velocities according to the most general Lagrangian [18]. However, notice that actually this should not be the case for momenta of order $2m_Q v$ of the gluonic field present in the covariant derivative. In fact, only such high-energy modes would survive, corresponding to the physical situation on which we are focusing, i.e. heavy quark-antiquark pair annihilation or creation.

From Eq. (26), the heavy quark-gluon coupling for an annihilation process is given by

$$\mathcal{L}_{coupling}^{(-+)} \quad = e^{-i2m_Q v\cdot x}\, gT_a A_\mu^a \times$$
$$\begin{array}{lll}
[& \bar{H}_v^{(-)}\, v^\mu\, h_v^{(+)} & (a) \\
- & \bar{h}_v^{(-)}\, v^\mu\, H_v^{(+)} & (b) \\
+ & \bar{h}_v^{(-)}\, \gamma_\perp^\mu\, h_v^{(+)} & (c) \\
+ & \bar{H}_v^{(-)}\, \gamma_\perp^\mu\, H_v^{(+)}] & (d)
\end{array} \tag{28}$$

There is an equivalent expression coming from $\mathcal{L}_{coupling}^{(+-)}$ just by means of the substitution $v\to -v$, corresponding to a heavy quark-antiquark pair creation process.

Let us make an important remark concerning the gluonic field. As can be seen at once, the exponential factor $e^{-i2m_Q v\cdot x}$ in Eq. (28) cancels against the strong x dependence of the A_μ^a field creating a hard gluon. Therefore we can write

$$A_\mu^{soft}(x) = e^{-i2m_Q v\cdot x}\, A_\mu(x) \tag{29}$$

Therefore, once removed the strong dependence of the gluon momentum on (twice) the heavy quark mass, we are left with an effective gluonic field A_μ^{soft}, with a mild dependence on the residual momentum, similarly as for heavy quark effective fields. Thus, the physical description of the annihilation process shown in Fig. 1 could be performed in terms of massless quarks and a soft gluonic field. This is pretty similar to the situation reached in heavy-light systems, constituting a cornerstone in our approach to be developed in the following Sections, providing a theoretical basis for the use of HQEFT to describe heavy quarkonium production.

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[3] Superscript "$(+)/(-)$" on the effective fields labels the particle/antiparticle (i.e. heavy quark/antiquark) sector of the theory [18]. Actually $\bar{h}_v^{(+)}$ ($\bar{h}_v^{(-)}$) corresponds to negative (positive) frequencies associated with creation (annihilation) operators of quarks (antiquarks). In fact some extra "$+/-$" signs should be added on the conjugate fields, i.e. $\bar{h}_v^{(+)-}$ and $\bar{H}_v^{(-)+}$, which however will be omitted to shorten the notation.
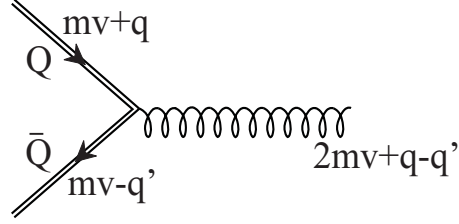
Figure 1: Heavy quark-antiquark pair annihilation into a gluon of momentum $2m_Q v + q - q'$. In the center-of mass system $q = (q_0, \mathbf{q})$ and $q' = (-q_0, \mathbf{q})$. This process can be described meaningfully by HQEFT if $\mathbf{q^2} << m_Q^2$, i.e. the square invariant mass of the gluon is close to $4m_Q^2$.

Next we want to eliminate the unwanted degrees of freedom associated to the "small" components $H_v^{(\pm)}$ in (a), (b) and (d) in Eq. (28). Notice that the piece labeled as (c) is the leading one in the above development.

5 Derivation of the annihilation vertex

In all our later development we shall assume almost free quarks in the initial (or final) state (see Fig. 1). Hence, heavy quark fields appearing on the external legs of the Feynman diagram should be solutions of the unperturbed Dirac equation of motion. Hence we can write

$$H_v^{(+)} = (iv\cdot\overrightarrow{\partial} + 2m_Q - i\epsilon)^{-1} i\overrightarrow{\partial}_\perp h_v^{(+)} \tag{30}$$

$$H_v^{(-)} = (-iv\cdot\overrightarrow{\partial} + 2m_Q - i\epsilon)^{-1} i\overrightarrow{\partial}_\perp h_v^{(-)} \tag{31}$$

and for the conjugate fields

$$\bar{H}_v^{(+)} = \bar{h}_v^{(+)} i\overleftarrow{\partial}_\perp (iv\cdot\overleftarrow{\partial} - 2m_Q + i\epsilon)^{-1} \tag{32}$$

$$\bar{H}_v^{(-)} = \bar{h}_v^{(-)} i\overleftarrow{\partial}_\perp (-iv\cdot\overleftarrow{\partial} - 2m_Q + i\epsilon)^{-1} \tag{33}$$

Let us note that we are using the free particle equations which can be viewed as derived from the non-interacting parts of the Lagrangians $\mathcal{L}^{(++)}$ and $\mathcal{L}^{(--)}$ respectively, i.e. we neglect the soft gluon interaction among heavy quarks amounting to describe them as plane waves, i.e. actually no bound states as a first approximation [19].

In momentum space we will write

$$u(m_Q v + q) = \left[1 + \frac{\slashed{q}_\perp}{2m_Q + v\cdot q} \right] u_h(q) \tag{34}$$

$$\overline{v}(m_Q v - q') = \overline{v}_h(-q') \left[1 - \frac{\slashed{q}'_\perp}{2m_Q - v\cdot q'} \right] \tag{35}$$

where $u(p)$ denotes the full QCD spinor whereas $u_h(q)$ represents the HQET spinor i.e. obeying $\slashed{v} u_h = u_h$; similarly $v(p)$ denotes the full QCD antispinor and $v_h(-q')$) represents the HQET antispinor obeying $\slashed{v} v_h = -v_h$.

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Actually, in order to arrive to Eqs. (34-35) from Eqs. (30-33) denominators have to be expanded as power series of derivatives acting on the x-dependent factors, assumed exponentials, to be finally resummed as geometric series of ratio $v{\cdot}q/2m_Q = -v{\cdot}q'/2m_Q = -q^2/4m_Q^2$ according to the on-shell conditions (5) and (16). As a consequence, Eqs. (34-35) are only valid under the condition $-q^2 < 4m_Q^2$, which implies $\mathbf{q^2} < 8m_Q^2$. Therefore, the requirement $\mathbf{q^2} < m_Q^2$ satisfies the above condition, allowing a non-relativistic expansion.

From the $\mathcal{L}^{(-+)}$ Lagrangian we readily get for the on-shell heavy quark (vector) current coupling to a gluon, suppressing color indices and matrices,

$$\bar{v}_h \left[\gamma_\perp^\mu + \frac{\slashed{q}'_\perp - \slashed{q}_\perp}{2m_Q + v{\cdot}q} v^\mu + \frac{\slashed{q}'_\perp \gamma^\mu \slashed{q}_\perp}{(2m_Q + v{\cdot}q)^2} \right] u_h \tag{36}$$

which also can be written as

$$\bar{v}_h \left[\gamma_\perp^\mu + \frac{i\sigma^{\mu\nu}(q'_\perp - q_\perp)_\nu}{2m_Q + v{\cdot}q} + \frac{\slashed{q}'_\perp \gamma^\mu \slashed{q}_\perp}{(2m_Q + v{\cdot}q)^2} \right] u_h \tag{37}$$

since [14]

$$P_- \left(v^\mu \gamma^\nu \right) P_+ \;=\; P_- \left(i\sigma^{\mu\nu} \;+\; \gamma^\mu v^\nu \right) P_+$$

with the projectors $P_\pm = (1\pm\slashed{v})/2$, and $q'_{\perp\nu}v^\nu = q_{\perp\nu}v^\nu = 0$.

Equations (36-37) are Lorentz covariant and hence valid in any reference frame. In the following Section we focus on the center-of-mass system, where some simplifications occur.

5.1 Center-of-mass frame

We shall make use of the anticommutation relation:

$$\slashed{q}'_\perp \gamma^\mu \slashed{q}_\perp = 2q'^\mu_\perp \slashed{q}_\perp - \gamma^\mu \slashed{q}'_\perp \slashed{q}_\perp$$

Now, in the center-of-mass frame we can write $\slashed{q}'_\perp \slashed{q}_\perp = -\mathbf{q^2}$, leading to the expression for the heavy quark vector current

$$\bar{v}_h \left[\frac{2E_q}{E_q + m_Q} \gamma_\perp^\mu + \frac{2q'^\mu_\perp \slashed{q}_\perp}{(E_q + m_Q)^2} \right] u_h \tag{38}$$

where $E_q = m_Q + q^0 = \sqrt{m_Q^2 + \mathbf{q^2}}$. Moreover, the following identity is satisfied (in the Dirac representation):

$$P_- \slashed{q}_\perp P_+ = \begin{pmatrix} 0 & 0 \\ \sigma{\cdot}\mathbf{q} & 0 \end{pmatrix} \tag{39}$$

Therefore identifying $u_h = \sqrt{E_q + m_Q}\, \xi$, and $v_h = \sqrt{E_q + m_Q}\, \eta$, we obtain from (38-39),

$$\eta^\dagger \left[2E_q\, \sigma^j + \frac{2q^j\, \sigma{\cdot}\mathbf{q}}{(E_q + m_Q)} \right] \xi \tag{40}$$

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i.e. the vertex obtained from full QCD in terms of the Pauli spinors ξ and η, allowing a systematic non-relativistic expansion: the leading term $2m(\eta^\dagger \sigma \xi)$ associated with (c) in Eq. (28) as expected.

Had we started from the Lagrangian $\mathcal{L}^{(+-)}$, i.e. focusing on a creation process, the heavy quark vector current would be

$$\overline{u}_h \left[\frac{2E_q}{E_q + m_Q} \gamma^\mu_\perp + \frac{2q'^\mu_\perp \slashed{q}_\perp}{(E_q + m_Q)^2} \right] v_h \tag{41}$$

and since

$$P_+ \slashed{q}_\perp P_- = \begin{pmatrix} 0 & -\sigma \cdot \mathbf{q} \\ 0 & 0 \end{pmatrix} \tag{42}$$

we rapidly recover

$$\xi^\dagger \left[2E_q \, \sigma^j - \frac{2q^j \, \sigma \cdot \mathbf{q}}{(E_q + m_Q)} \right] \eta \tag{43}$$

which is the analogous expression to Eq. (A.11b) in the c.o.m. frame as given in [20].

In sum, we have derived in this Section the vector current of on-shell heavy quarks coupling to a hard gluonic field directly from the corresponding Lagrangian $\mathcal{L}^{(-+)}$, $\mathcal{L}^{(+-)}$ pieces, giving consistency to our formalism.

6 Heavy Quarkonia Hadroproduction

In 1992-93, CDF and D0 Collaborations surprisingly found an excess of charmonia prompt hadroproduction at the $p\overline{p}$ Tevatron collider [21] w.r.t. to the theoretical expectations based on what was considered at that time as conventional wisdom, the so-called color-singlet model (CSM) [22]. This disagreement was particularly amazing because data spread over large transverse momenta where the theoretical analysis ought to be quite clean. Therefore another mechanism, known as the color-octet model (COM), was proposed by Braaten and Fleming in 1995 [23], superseding (generalizing actually) the CSM. Soon later, the COM was viewed as deriving from NRQCD [6], giving an adequate framework for the factorization of the inclusive production cross section into short- and long-distance parts, and providing a sound basis to the theoretical analysis so far done.

However, current problems in the interpretation of charmonia hadroproduction [30], especially regarding the (expected but unobserved) transverse resonance polarization, have cast serious doubts on the validity of NRQCD when applied to these processes. Actually, the meaningful application of the effective theory relies on the presumed convergence of the expansion governed by the typical velocity of heavy quarks v' and the strong coupling constant α_s. Different contributions are assumed as leading whereas others are neglected under the assumption of the validity of the velocity scaling rules derived from NRQCD, mainly based on dimensional counting and very general physical arguments.

In this context some authors [15] have recently hypothesized that the correct power counting for charmonia should be the dimensionless parameter Λ_{QCD}/m_c, along the lines of HQET [4], leading to predictions which could differ from the expectations coming from the usual velocity scaling rules of NRQCD. In the following and based on our previous development, we want to give a theoretical support to such a point of view. In fact we are advocating in this paper that HQEFT could provide an appropriate and consistent framework for the analysis of hadroproduction of heavy quarkonia. To this end, from Fig. 1 we may identify the heavy quark three-momentum $\mathbf{q}$ relative to the c.o.m. as the residual three-momentum $\mathbf{k}$ introduced in Section 2.

On the other hand, in charmonium systems it seems sensible to assume $v' \simeq |\mathbf{q}|/m_Q \simeq \Lambda/m_Q$, where Λ stands for a "typical" scale characterizing the quarkonium soft dynamics, of order few times Λ_{QCD}. Therefore dynamical (i.e. non-Coulombic) gluons in $c\bar{c}$ bound states should be of the type (Λ, Λ) though the typical residual four-momenta of bound heavy quarks should be of the type $(\Lambda^2/2m_Q, \Lambda)$ (see [24, 15]), in close analogy with heavy-light hadrons. Hence the heavy quark field expansions given in Eqs. (12) and (18) about the heavy quark mass, and the reinterpretation of $e^{-i2m_Q v \cdot x} A^\mu$ in Eq. (29) as a soft gluonic field coupling to a heavy quark vector current make sense altogether.

On the other hand, the effective QCD Lagrangian [6] to deal with heavy quarkonium is usually written as:

$$\mathcal{L}_{eff} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_{light} \tag{44}$$

where $\mathcal{L}_2$ contains bilinear operators, $\mathcal{L}_4$ stands for the four-fermion piece and $\mathcal{L}_{light}$ takes into account light quarks and gluons.

In NRQCD heavy quarks are treated according to a Schrödinger-type field theory with separate two-component Pauli spinors (commonly denoted as ψ and χ), rather than with four-component Dirac fields as in HQEFT. Relativistic effects of full QCD are reproduced through correcting terms bilinear in the quark field or in the antiquark field. However, mixed two-fermion operators involving both quark and antiquark fields in $\mathcal{L}_2$ corresponding to creation/annihilation of $Q\overline{Q}$ pairs are *excluded*.

On the other hand, in order to account for heavy quarkonium annihilation or creation, a set of four-fermion operators are thereby included in the piece $\mathcal{L}_4$ of the effective Lagrangian (44), representing local interactions whose short-distance coefficients have to be determined by means of a matching procedure to QCD as described, for instance, in [20].

Actually, in this work we are advocating that the $\mathcal{L}_2$ piece of the effective Lagrangian can be extended as

$$\mathcal{L}_2 = \mathcal{L}_2^{(++)} + \mathcal{L}_2^{(--)} + \mathcal{L}_2^{(-+)} + \mathcal{L}_2^{(+-)} \tag{45}$$

where the $\mathcal{L}_2^{(-+)}$ and $\mathcal{L}_2^{(-+)}$ terms can be identified with those coming from Eqs. (26-27), therefore taking into account $Q\overline{Q}$ pair formation and annihilation at tree-level.

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[4] The authors of [15] introduce the acronym NRQCD_c which we identify with HQEFT applied to charmonium.
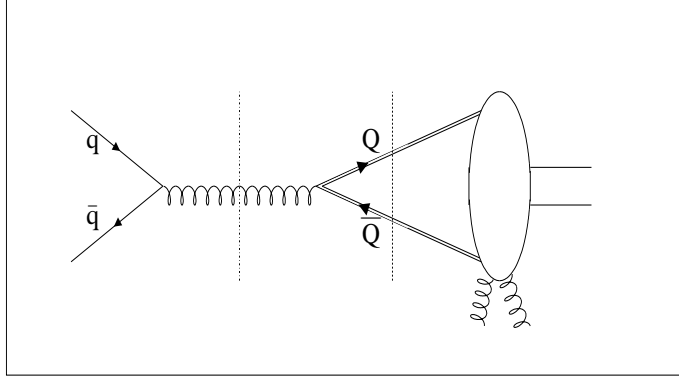
Figure 2: Graph representing a $q\overline{q}$ annihilation followed by the creation of a $Q\overline{Q}[^3S_1]$ pair, evolving into a final spin-triplet quarkonium color-singlet state by emission of two soft gluons via a double chromoelectric dipolar transition. The two dotted lines delimit the gluon-quark vertex to be described along the lines of HQEFT following our suggestion of including the description of the $Q\overline{Q}$ creation within the realm of HQEFT. Moreover, local four-fermion terms in the Lagrangian accounting for heavy quarkonium production can be thought as derived from the $\mathcal{L}^{(+-)}$ and $\mathcal{L}^{(-+)}$ pieces of the same HQEFT Lagrangian.

Notice that, as far as the heavy quark mass dependence has been removed from the effective fields, only low-energy modes (of the order of Λ around m_Q,) remain as stressed before. Hence the situation turns out to be very similar to the standard application of HQEFT to heavy-light systems. On the other hand, we keep the $\mathcal{L}_4$ piece in the effective Lagrangian, although written in terms of HQEFT fields.

For definiteness, let us consider quarkonium hadroproduction through the partonic channel:

$$q\overline{q} \rightarrow g^* \rightarrow (Q\overline{Q})_8[^3S_1] \rightarrow H \ X$$

where H denotes a particular quarkonium state. (The diagrammatic representation for this production process yielding a vector resonance is shown in Fig. 2.)

6.1 Color-octet matrix elements from HQEFT

Under the assumption of factorization, the cross section can be written for inclusive resonance production as [6]

$$d\sigma(H + X) \ = \ \sum_n d\hat{\sigma}[(Q\overline{Q})_n + X] \ < \mathcal{O}_n^H > \tag{46}$$

where $d\hat{\sigma}$ stands for the production cross section of a $Q\overline{Q}$ pair in a definite color and angular-momentum state labeled by n. The short-distance interaction, represented by $d\hat{\sigma}$ in (46), can be accounted for by perturbative QCD, whereas the long-distance process is encoded in the vacuum expectation values of four-fermion local operators, i.e. $< \mathcal{O}_n^H >$ [6]. According to NRQCD, the relative importance of the various terms in the factorization is determined by the order in α_s and some dimensionless ratios of kinematic variables in the short-distance factors, and by a hierarchy based on the order in v' of the matrix elements. Hence, the velocity scaling of the MEs is determined

by the number of derivatives in the respective operators and the number of electric and magnetic dipole transitions needed to reach the final physical quarkonium state from the initial $Q\overline{Q}$ pair produced at the short-distance process.

Pictorially, the long-distance evolution lies on the right of the rightmost dotted line in Fig. 2, belonging to the realm of the effective Lagrangian $\mathcal{L}_4$ and providing the probability for a heavy quark-antiquark pair to evolve into a (definite) final quarkonium state. On the other hand, the subprocess $g^* \to Q\overline{Q}$ can be ascribed to the piece $\mathcal{L}_2^{(+-)}$ of the effective Lagrangian from our viewpoint. Therefore we shall use the $h_v^{(+)}$ and $h_v^{(-)}$ HQEFT fields when rewriting the relevant operators. For example, the $O_8(^3S_1)$ 6-dimensional operator [6] can be expressed in the form of a current×current term (see figure 3), i.e.

$$O_8(^3S_1) \; = \; \bar{h}_v^{(-)} \, \gamma_\perp^\mu \, T_a \, h_v^{(+)} \, (a_H^\dagger a_H) \, \bar{h}_v^{(+)} \, \gamma_{\perp\mu} \, T_a \, h_v^{(-)} \tag{47}$$

where $a_H(a_H^\dagger)$ stands for annihilation (creation) operators of heavy quarkonium H, such that

$$a_H^\dagger a_H \; = \; \sum_X |H+X><H+X| \tag{48}$$

The above $O_8(^3S_1)$ operator can be seen as deriving from the corresponding piece of the $\mathcal{L}_4$ Lagrangian [6] now expressed in terms of HQEFT fields,

$$\mathcal{L}_4^{^3S_1^{[8]}} \; = \; \frac{F_8(^3S_1)}{m_Q^2} \; \bar{h}_v^{(+)} \gamma_\perp^\mu \, T_a \, h_v^{(-)} \, \bar{h}_v^{(-)} \gamma_{\perp\mu} \, T_a \, h_v^{(+)} \tag{49}$$

The physical interpretation of (47) is that of a local four-fermion operator which creates a slightly off-shell $Q\overline{Q}$ pair in a $^3S_1^{[8]}$ state and destroys it again, analogously as in NRQCD. Its vacuum expectation value $< 0|O_8(^3S_1)|0 > \; \equiv \; < O_8(^3S_1) >$ corresponds to the usual color-octet matrix element of NRQCD. Notice, however, that the current×current structure can be thought now as deriving from the $\mathcal{L}_2^{(-+)}$ and $\mathcal{L}_2^{(+-)}$ Lagrangian pieces, involving only low-frequency modes and providing consistency to all the theoretical framework. High-energy modes are encoded in the short-distance coefficients ($F_8(^3S_1)$ in this particular case), as usual in low-energy effective theories. (Nevertheless, the matching procedure, although formally the same as in NRQCD, might be seen as "built in" because the effective fields in the four-quark local operators follow from $\mathcal{L}_2$.)

Hence the power counting of the effective theory should follow the lines of HQEFT, as far as $m_Q v' \simeq \Lambda$, and the expansion parameter can be identified as Λ/m_Q. However, let us remark that the intermediate colored bound state $(Q\overline{Q})_8$ evolving into final charmonium actually is a kind of *hybrid* state [25], whose "size" (and even nature) is not accurately known. Therefore, one could expect Λ to be of order few times Λ_{QCD}, namely 700-800 MeV, and typically $(\Lambda/m_Q)^2 \simeq 1/4$, but with large uncertainties.
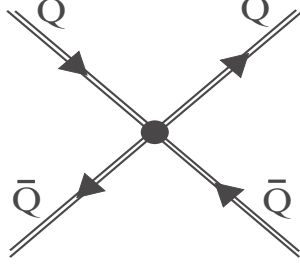
Figure 3: Since the formation of the quarkonium state H takes place over distances of the order of $1/\Lambda$, the use of HQEFT is justified. Local interactions give rise to the $O_8(^3S_1)$ operator among others, in terms of $h_v^{(\pm)}$ fields coming from the $\mathcal{L}_2$ piece of the Lagrangian.

6.2 Power scaling rules

Under the assumption that HQEFT could be a suitable effective theory to deal with charmonia production, we will follow a reasoning analogous to [15, 24], arguing that there could be a hierarchy of the matrix elements, different from "standard" NRQCD. Therefore we expect that a single chromoelectric dipole (E1) transition should scale as Λ/m_Q *at the amplitude level*. Since two E1 transitions are required to reach a $^3S_1^{[1]}$ color-singlet state from a $^3S_1^{[8]}$ color-octet one, the resulting suppression factor is $(\Lambda/m_Q)^2$. On the other hand, a single chromomagnetic transition (M1) should scale as Λ/m_Q, because of the time derivative on the gluonic field strength-tensor, picking up a typical energy of order Λ. Thus, *at the cross section level*, the power countings are expected to be of order $(\Lambda/m_Q)^4$ and $(\Lambda/m_Q)^2$ for a double E1 transition and a single M1 transition, respectively.

Now, defining as usual the linear combination of MEs [5] (where r is an adjustable parameter varying between 3 and 3.5 for charmonia hadroproduction) as

$$M_r^{\psi(nS)} \; = \; < O_8^{\psi(nS)}(^1S_0) > \; + \; r \; \frac{< O_8^{\psi(nS)}(^3P_J) >}{m_Q^2} \tag{50}$$

the former one, i.e. $< O_8^{\psi(nS)}(^1S_0) >$, should become leading in the overall power counting of $M_r^{\psi(nS)}$.

Let us now define the ratio

$$R^{\psi(nS)} \; = \; \frac{M_r^{\psi(nS)}}{< O_8^{\psi(nS)}(^3S_1) >} \quad ; \quad (n = 1, 2) \tag{51}$$

Since HQEFT predicts a typical scaling (relative to the color-singlet contribution) of $(\Lambda/m_Q)^4$ for $< O_8^{\psi(nS)}(^3S_1) >$, and $(\Lambda/m_Q)^2$ for $M_r^{\psi(nS)}$, one should expect

$$R^{\psi(nS)} \sim \left(\frac{m_Q}{\Lambda}\right)^2 \tag{52}$$

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[5]The little difference in shape between the $^1S_0^{[8]}$ and $^3P_J^{[8]}$ contributions as a function of transverse momentum does not permit independent fits and only a linear combination of them can be extracted from experimental data [29, 28].

which could also be expressed as $1/v'^2$ in terms of the more conventional power counting based on the typical velocity of heavy quarks, under the assumption that $v' \simeq \Lambda/m_Q$.

Conversely, in standard NRQCD both matrix elements $< O_8^{\psi(nS)}(^3S_1) >$ and $M_r^{\psi(nS)}$, should scale similarly (as v'^4 w.r.t. to the leading color-singlet component). Therefore no hierarchy should appear, and one would expect $R^{\psi(nS)} \simeq 1$.

In the following we carry out a check based on the available experimental information [6] extracted by several authors from fits to Tevatron data on charmonia hadroproduction. In tables 1 and 2 we show the values of the color-octet matrix elements $< O_8^{\psi(nS)}(^3S_1) >$ and $M_r^{\psi(nS)}$ for J/ψ and ψ' respectively, and the corresponding ratios $R^{\psi(nS)}$.

Table 1: Values (in units of 10^{-3} GeV^3) of $< O_8^{\psi(1S)}(^3S_1) >$ and $M_r^{\psi(1S)}$ matrix elements (r varies between 3 and 3.5) and their ratios $R^{\psi(1S)}$, obtained from Tevatron data on prompt J/ψ inclusive hadroproduction. Error bars are only statistical.

ME:	$< O_8^{\psi(1S)}(^3S_1) >$	$M_r^{\psi(1S)}$	$R^{\psi(1S)}$
[26]	11.9±1.4	45.4±11.1	3.8±1.0
[27]	10.6±1.4	43.8±11.5	4.1±1.2
[28]	3.3±0.5	14.4±2.8	4.4±1.1
[29]	6.6±2.1	66±5	10.0±3.3
[30]	3.9±0.7	66±7	16.9±3.5

Table 2: Values (in units of 10^{-3} GeV^3) of $< O_8^{\psi(2S)}(^3S_1) >$ and $M_r^{\psi(2S)}$ matrix elements (r varies between 3 and 3.5) and their ratios $R^{\psi(2S)}$, obtained from Tevatron data on prompt ψ' inclusive hadroproduction. Error bars are only statistical.

ME:	$< O_8^{\psi(2S)}(^3S_1) >$	$M_r^{\psi(2S)}$	$R^{\psi(2S)}$
[30]	3.7±0.9	7.8±3.6	2.1±1.1
[28]	1.4±0.3	3.3±0.9	2.4±0.8
[26]	5.0±0.6	18.9±4.6	3.8±1.0
[29]	4.6±1.0	17.7±5.7	3.8±1.5
[27]	4.4±0.8	18.0±5.6	4.1±1.5

We realize that indeed a hierarchy seems to exist for $< O_8^{\psi(nS)}(^3S_1) >$ and $M_r^{\psi(nS)}$. Although $R^{\psi(1S)}$ varies in a somewhat wide range, a value equal or greater than *four* seems to be favored. On the other hand, the ratio $R^{\psi(2S)}$ turns out to be slightly but systematically smaller than $R^{\psi(1S)}$, but close to *four* as well, i.e. compatible with the expectation $v'^2 \simeq \Lambda^2/m_Q^2 \simeq 1/4$, already mentioned at the end of Section 6.1.

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[6]We limit ourselves to those references where MEs extractions of *both* J/ψ and ψ' are given; notice that distinct theoretical inputs may have been used, as for example different parton distribution functions.

In fact one should bear in mind that $tR^{\psi(nS)}$ is not a fixed quantity for the whole charmonium family (nor the velocity v'). This remark is in agreement with the expected larger value of the typical velocity v' for higher n states in the charmonium sector or, equivalently, a smaller ratio m_Q/Λ.

Another source for the determination of long-distance matrix elements comes from fixed-target experiments. In Ref. [31] Beneke and Rothstein find $M_7^{\psi(1S)} = 30$, having fixed $< O_8^{\psi(2S)}(^3S_1) >$ equal to 6.6 (in units of GeV^{-3}). For the ψ' resonance, the numerical values are $M_7^{\psi(2S)} = 5.2$ having fixed $< O_8^{\psi(1S)}(^3S_1) >$ equal to 4.6 (also in units of GeV^{-3}) [7]. Therefore one gets $R^{\psi(1S)} = 4.5$ and $R^{\psi(2S)} = 1.1$, indicating again a smaller value for the $2S$ state, although subject to large uncertainties too [8].

NRQCD matrix elements can also be determined from inclusive decays of B mesons. In a recent work [33], Ma obtains a combination of $< O_8^{\psi(nS)}(^1S_0) >$ and $< O_8^{\psi(nS)}(^3P_J) >$ MEs in the framework of HQEFT, having fixed $< O_8^{\psi(nS)}(^3S_1) >$ equal to $10.6(4.4)\cdot10^{-3}$ GeV^3 for the J/ψ and ψ' respectively. The resulting ratios (not exactly the same as in tables 1 and 2 since r takes the value of 1.13) are $R^{\psi(1S)} \simeq R^{\psi(2S)} \simeq 2.3$ which, although slightly smaller than those presented in the tables, are not inconsistent with them in view of the smaller r value and the uncertainties involved in the extraction [33].

7 Summary and last remarks

In this paper we have carefully reviewed all steps to derive from the full QCD tree-level Lagrangian a complete transformed Lagrangian in terms of the heavy quark effective fields $h_v^{(\pm)}$, keeping the particle-antiparticle mixed pieces allowing for heavy quark-antiquark pair annihilation/creation. Let us note that such pieces are not generally neither used nor shown in similar developments in the literature, with a few exceptions [13, 14, 8, 9].

Indeed, it may seem quite striking that a low-energy effective theory could be appropriate to deal with hard processes such as $Q\overline{Q}$ annihilation or creation. The keypoint is that, assuming a kinematic regime where heavy quarks/antiquarks are almost on-shell and moving with small relative momentum, the strong momentum dependence associated with the heavy quark masses can be removed, so that a description based on the low frequency modes of the fields still makes sense, as in HQET applied to heavy-light hadrons. Such a kinematic regime can be well matched by heavy quarkonia and intermediate colored $(Q\overline{Q})_8$ bound states predicted by the COM.

In particular, we have focused on an annihilation process with initial-state quarks satisfying the Dirac equation of motion for free fermions. Thus, we have derived directly from the $\mathcal{L}^{(-+)}$ piece the heavy quark vector current coupling to a background

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[7]In fixed-target experiments it is not possible to fit simultaneously both matrix elements because they display a similar shape as a function of the beam energy.

[8]In Ref. [32] smaller values for $M_7^{\psi(nS)}$ are obtained as the inclusion of NLO corrections tend to decrease the values needed for the MEs. However, all values shown in tables 1 and 2 refer to LO extractions so we prefer not mixing them.

gluonic field, recovering a well known expression shown in the literature [20] allowing a non-relativistic expansion in the matching procedure of NRQCD and full QCD.

In the second part of this work we have examined the meaning of using a HQEFT Lagrangian to account for heavy quarkonium production. Thereby we kept the $\mathcal{L}^{(-+)}$ and $\mathcal{L}^{(+-)}$ pieces in the effective Lagrangian, from which the $\mathcal{L}_4$ four-fermion piece can be "constructed". The result looks formally the same as in conventional NRQCD although the fields are the HQEFT ones, i.e. $h_v^{(\pm)}$, giving self-consistency to all the framework.

From a phenomenological point of view, we advocate the possibility of using the extended version of HQEFT for describing inclusive hadroproduction of charmonium states according to the color-octet mechanism. Indeed, the four different energy scales usually invoked in NRQCD (i.e. m_Q, $m_Q v'$, $m_Q v'^2$ and Λ_{QCD}) basically reduce to two in HQEFT: m_Q and a typical hadronic scale, Λ, of the order of a few Λ_{QCD}'s, in a reasonable accordance with the dynamics of charmonium systems. Conversely, inclusive production of bottomonium resonances should likely be better described by NRQCD because of the clearer difference among the scales.

From inspection of tables 1 and 2, we can conclude that there are hints indicating that HQEFT may be considered as a candidate for describing J/ψ and ψ' hadroproduction - although likely in a limited way because of the approximations in the scales and the lack of a static limit *à la HQET* when $\Lambda/m_Q \to 0$, as stressed elsewhere [5, 15]. Hence we give support to the hypothesis presented in Ref. [15] of basing the power counting for charmonium hadroproduction along the lines of HQEFT, to some extent.

As a final comment, let us remark that still the uncertainties in the phenomenological extractions of the color-octet matrix elements (choice of numerical values for the heavy quark mass and renormalization/fragmentation scales, parton distribution functions, etc) are quite large. Moreover, new approaches have recently appeared (e.g. the k_T factorization [34, 35]) while the experimental evidence is not yet conclusive to decide which framework should be the most adequate to describe hadroproduction of heavy resonances.

Acknowledgements

References

[1] M. Neubert, hep-ph/0001334.

[2] B. Grinstein, *Annu. Rev. Nucl. Part. Sci.*, **42**, 101 (1992).

[3] M. Neubert, *Phys. Rep.* **245**, 259 (1994).

[4] M.A. Sanchis-Lozano, *Phys. Lett.* **B312**, 333 (1993).

[5] M.A. Sanchis-Lozano, *Nucl. Phys.* **B440**, 251 (1995) and references therein.

[6] G.T. Bodwin, E. Braaten and G.P. Lepage, *Phys. Rev.* **D51**, 1125 (1995).

[7] G.P. Lepage and B.A. Thacker, *Nucl. Phys. B, (Proc. Suppl.)* **4**, 199 (1988).

[8] Y.A. Yan, Y.L. Wu and W.Y. Wang, *Int. J. Mod. Phys.* **A15**, 2735 (2000).

[9] Y.L. Wu and Y.A. Yan, *Int. J. Mod. Phys.* **A16**, 285 (2001).

[10] F. Berto and M.A. Sanchis-Lozano, hep-ph/0010091.

[11] M.A. Sanchis-Lozano, *Nuov. Cim.* **A110**, 295 (1997), hep-ph/9612210.

[12] M.A. Sanchis-Lozano, hep-ph/9710408.

[13] Y.L. Wu, *Mod. Phys. Lett.* **A8**, 819 (1993).

[14] F. Berto and M.A. Sanchis-Lozano, *Nuov. Cim.* **A112**, 1181 (1999), hep-ph/9810549.

[15] S. Fleming, I.Z. Rothstein, A.L. Leibovich, hep-ph/0012062.

[16] J.F. Donaghue, E. Golowich, B.R. Holstein, *Dynamics of the Standard Model* (Cambridge University Press, 1992).

[17] T. Mannel, W. Roberts and Z. Ryzak, *Nucl. Phys.* **B368**, 204 (1992).

[18] H. Georgi, Phys. Lett. **B240**, 447 (1990).

[19] F. Hussain, J.G. Körner and G. Thompson, *Ann. Phys.* **206**, 334 (1991).

[20] E. Braaten and Y-Q Chen, *Phys. Rev.* **D55**, 2693 (1997); **D54**, 3216 (1996).

[21] CDF Collaboration, *Phys. Rev. Lett.* **69**, 3704 (1992); **79**, 572 (1997).

[22] G.A. Schuler, hep-ph/9403387.

[23] E. Braaten and S. Fleming, *Phys. Rev. Lett.* **74**, 3327 (1995).

[24] G.A. Schuler, *Int. J. Mod. Phys.* **A12**, 3951 (1997).

[25] B. Grinstein, hep-ph/9811264.

[26] P. Nason *et al.*, hep-ph/0003142.

[27] M. Beneke, hep-ph/9703429.

[28] B. Cano-Coloma and M.A. Sanchis-Lozano, *Nucl. Phys.* **B508**, 753 (1997).

[29] P. Cho and A.K. Leibovich, *Phys. Rev.* **D53**, 150 (1996); *ibid* **D53**, 6203 (1996).

[30] E. Braaten, B.A. Kniehl and J. Lee, *Phys. Rev.* **D62**, 094005 (2000); B.A. Kniehl and J. Lee, hep-ph/0007292.

[31] M. Beneke and I.Z. Rothstein, *Phys. Rev.* **D54**, 2005 (1996); *ibid* **D54**, 7082(E) (1996).

[32] F. Maltoni, hep-ph/0007003.

[33] J.P. Ma, *Phys. Lett.* **B488**, 55 (2000), hep-ph/0006060.

[34] Ph. Hägler *et al.*, *Phys. Rev. Lett.* **86**, 1446 (2001).

[35] F. Yuan and K-T Chao, *Phys. Rev.* **D63**, 034006 (2001).